PHYSICS - 1999

PART - A

Directions : *Select the most appropriate alternative a, b, c & d in questions 1-25*

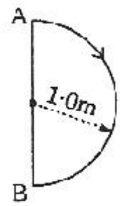
1. A closed compartment containing gas is moving with some acceleration in horizontal direction. Neglect effect of gravity. Then the pressure in the compartment is :

(A) same everywhere (B) lower in front side
(C) lower in rear side (D) lower in upper side.

2. The ratio of the speed of sound in nitrogen gas to that in helium gas at 300K is :

(A) $\sqrt{2/7}$ (B) $\sqrt{1/7}$
(C) $\sqrt{3}/5$ (D) $\sqrt{6}/5$

3. In 1.0S, a particle goes from point A to point B, moving in a semicircle (see figure). The magnitude of the average velocity is :

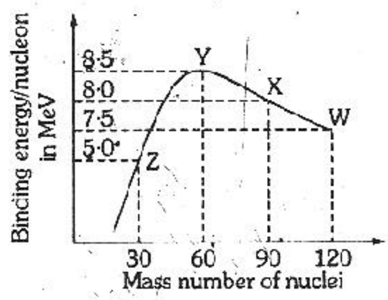


(A) 3.14 m/s (B) 2.0 m/s
(C) 1.0 m/s (D) zero

4. A charged particle is released from rest in a region of steady and uniform electric and magnetic fields which are parallel to each other. The particle will move in a :

(A) straight line (B) circle
(C) helix (D) cycloid

5. Binding energy per nucleon Vs mass number curve for nuclei is shown in figure. W, X, Y and Z are four nuclei indicated on the curve. The process that would release energy is :



(A) $Y \rightarrow 2Z$
(B) $W \rightarrow X + Z$
(C) $W \rightarrow 2Y$
(D) $X \rightarrow Y + Z$

6. Order of magnitude of density of uranium nucleus is ($m_p = 1.67 \times 10^{-27}$ kg) :

(A) 10^{20} kg/m^3 (B) 10^{17} kg/m^3
(C) 10^{14} kg/m^3 (D) 10^{11} kg/m^3

7. Two identical circular loops of metal wire are lying on a table without touching each other. Loop A carries a current which increases with time. In response, the loop B :

(A) remains stastionery
(B) is attracted by the loop A
(C) is repelled by the loop A
(D) rotates about its CM, with CM fixed.

8. A spring of force constant K is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force constant of :

(A) 2/3 K (B) 3/2 K

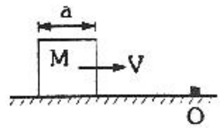
(C) 3 K (D) 6 K

9. ^{22}Ne nucleus, after absorbing energy, decays into two α-particles and an unknown nucleus. The unknown nucleus is :

(A) nitrogen (B) carbon

(C) boron (D) oxygen.

10. A cubical block of side a moving with velocity V on a horizontal smooth plane as shown. It hits a ridge at point O. The angular speed of the block after it hits O is :



(A) 3V/4a (B) 3V/2a

(C) $\sqrt{3}V/\sqrt{2}a$ (D) zero

11. Yellow light is used in a single slit diffraction experiment with slit width of 0·6 mm. If yellow light is replaced by X-rays, then the observed pattern will reveal :

(A) that the central maximum is narrower

(B) more number of fringes

(C) less number of fringes

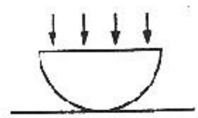
(D) no diffraction pattern

12. Two identical metal plates are given positive charges Q_1 and Q_2 ($< Q_1$) respectively. If they are now brought close together to form a parallel plate capacitor with capacitance C, the potential difference between them is :

(A) $(Q_1 + Q_2)/2C$ (B) $(Q_1 + Q_2)/C$

(C) $(Q_1 - Q_2)/C$ (D) $(Q_1 - Q_2)/2C$

13. A thin slice is cut out of a glass cylinder along a plane parallel to its axis. The slice is placed on a flat plate as shown. The observed interference fringes from this combination shall be :



(A) straight

(B) circular

(C) equally spaced

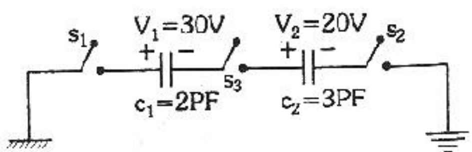
(D) having fringe spacing which increases as we go outwards.

14. A coil of inductance 8.4 mH and resistance 6Ω is connected to a 12V battery. The current in the coil is 1.0A at approximately the time :

(A) 500 s (B) 20 s

(C) 35 ms (D) 1 ms

15. For the circuit shown, which of the following statements is true :



(A) With S_1 closed, $V_1 = 15V, V_2 = 20V$

(B) With S_3 closed, $V_1 = V_2 = 25V$

(C) With S_1 and S_2 closed, $V_1 = V_2 = 0$

(D) With S_1 and S_3 closed, $V_1 = 30V$, $V_2 = 20V$

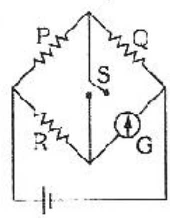
16. A concave lens of glass, refractive index 1.5 has both surfaces of same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as a :

(A) Convergent lens of focal length 3.5 R
(B) Convergent lens of focal length 3.0 R
(C) divergent lens of focal length 3.5 R
(D) divergent lens of focal length 3.0 R

17. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is :

(A) 4 RT (B) 15 RT
(C) 9 RT (D) 11 RT

18. In the circuit shown $P \neq R$, the reading of galvonometer is same with switch S open or closed. Then :



(A) $I_R = I_G$
(B) $I_P = I_G$
(C) $I_Q = I_G$
(D) $I_Q = I_R$

19. A smooth sphere A is moving on a frictionless horizontal plane with angular velocity ω and centre of mass velocity v. It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision, their angular speeds are ω_A and ω_B respectively. Then :

(A) $\omega_A < \omega_B$ (B) $\omega_A = \omega_B$
(C) $\omega_A = \omega$ (D) $\omega_B = \omega$

20. In hydrogen spectrum the wavelength of H_α line is 656 nm; whereas in the spectrum of a distant galaxy H_α line wavelength is 706 nm. Estimated speed of galaxy with respect to earth is :

(A) 2×10^8 m/s (B) 2×10^7 m/s
(C) 2×10^6 m/s (D) 2×10^5 m/s

21. A particle free to move along the x-axis has potential energy given by $U(x) = K[1 - \exp(-x^2)]$ for $-\infty \le x \le +\infty$ where K is a positive constant of appropriate dimensions. Then :

(A) At points away from the origin, the particle is in unstable equilibrium
(B) For any finite non-zero value of x, there is a force directed away from the origin
(C) If its total mechanical energy is K/2, it has its minimum kinetic energy at the origin.
(D) For small displacements from x = 0, the motion is simple hormonic

22. A particle of mass M at rest decays into two particles of masses m_1 and m_2 having non-zero velocities. The ratio of the de-Broglie wavelengths of the particles λ_1/λ_2 is :

(A) m_1/m_2 (B) m_2/m_1
(C) 1.0 (D) $\sqrt{m_2}/\sqrt{m_1}$

23. A circular loop of radius R, carrying current I, lies in x-y plane with its centre at the origin. The total magnetic flux through x-y plane is :

(A) directly proportional to I (B) directly proportional to R

(C) directly proportional to R^2 (D) zero

24. Which of the following is a correct statement :

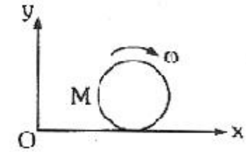
(A) Beta rays are same as cathode rays

(B) Gamma rays are high energy neutrons

(C) Alpha particles are singly ionized helium atoms

(D) Protons and neutrons have exactly the same mass.

25. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown. The magnitude of angular momentum of the disc about the origin O is :



(A) $\left(\frac{1}{2}\right) MR^2\omega$ (B) $MR^2\omega$

(C) $\left(\frac{3}{2}\right) MR^2\omega$ (D) $2\ MR^2\omega$

Directions : *Question numbers 26–35 carry 3 marks each and may have more than one correct answers. All correct answers must be marked to get any credit in these questions.*

26. The coordinates of a particle moving in a plane are given by $x(t) = a\cos(pt)$ and $y(t) = b\sin(pt)$ where a, b (< a) and p are positive constants of appropriate dimensions. Then :

(A) the path of the particle is an ellipse

(B) the velocity and acceleration of the particle are normal to each other at $t = \pi/2p$

(C) the acceleration of the particle is always directed towards a focus

(D) the distance travelled by the particle in time interval $t = 0$ to $t = \pi/2p$ is a.

27. The half-life period of a radioactive element X is same as the mean life time of another radioactive element Y. Initially both of them have the same number of atoms. Then :

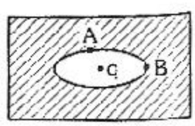
(A) X and Y have the same decay rate initially

(B) X and Y decay at the same rate always

(C) Y will decay at a faster rate than X

(D) X will decay at faster rate than Y

28. An elliptical cavity is carved within a perfect conductor. A positive charge q is placed at the centre of the cavity. The points A and B are on the cavity surface as shown in the figure. Then :



(A) electric field near A in the cavity = electric field near B in the cavity

(B) charge density at A = charge density at B

(C) potential at A = potential at B

(D) total electric field flux through the surface of the cavity is q/ϵ_0.

29. Three simple harmonic motions in the same direction having the same amplitude and same period are superposed. If each differ in phase from the next by 45°, then :

(A) the resultant amplitude is $(1 + \sqrt{2})\,a$

(B) the phase of the resultant motion relative to the first is 90°

(C) the energy associated with the resulting motion is $(3 + 2\sqrt{2})$ times the energy associated with any single motion

(D) the resulting motion is not simple harmonic

30. As a wave propagates :

(A) the wave intensity remains constant for a plane wave

(B) the wave intensity decreases as the inverse of the distance from the source for a spherical wave

(C) the wave intensity decreases as the inverse square of the distance from the source for a spherical wave

(D) total intensity of the spherical wave over the spherical surface centered at the source remains constant at all times.

31. A bimetallic strip is formed out of two identical strips one of copper and the other of brass. The coefficients of linear expansion of the two metals are α_C and α_B. On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then R is :

(A) proportional to ΔT

(B) inversely proportional to ΔT

(C) proportional to $|\alpha_B - \alpha_C|$

(D) inversely proportional to $|\alpha_B - \alpha_C|$

32. When a potential difference is applied across, the current passing through :

(A) an insulater at 0 K is zero

(B) a semiconductor at 0 K is zero

(C) a metal at 0 K is finite

(D) a p-n diode at 300 K is finite if it is reverse biased.

33. $Y(x, t) = \dfrac{0 \cdot 8}{[(4x + 5t)^2 + 5]}$ represents a moving pulse where x and y are in metres and t in second. Then :

(A) pulse is moving in positive x direction

(B) in 2s it will travel a distance of 2.5 m

(C) Its maximum displacement is 0.16 m

(D) it is a symmetric pulse

34. In a wave motion $y = a \sin (Kx - \omega t)$, y can represent :

(A) electric field

(B) magnetic field

(C) displacement

(D) pressure.

35. Standing waves can be produced :

(A) on a string clamped at both ends

(B) on a string clamped at one end and free at the other

(C) when incident wave gets reflected from a wall

(D) when two identical waves with a phase difference of π are moving in the same direction.

ANSWERS

1. B,	**2.** C,	**3.** B,	**4.** A,	**5.** C,	**6.** B,
7. C,	**8.** B,	**9.** B,	**10.** A,	**11.** D,	**12.** D,
13. A,	**14.** D,	**15.** D,	**16.** A,	**17.** D,	**18.** A,
19. C,	**20.** B,	**21.** D,	**22.** C,	**23.** D,	**24.** A,
25. C,	**26.** A, B, C,	**27.** C,	**28.** C, D,	**29.** A, C,	**30.** A, C, D,
31. B, D,	**32.** A, B, D,	**33.** B, C, D,	**34.** A, B, C, D,		**35.** A, B, C.

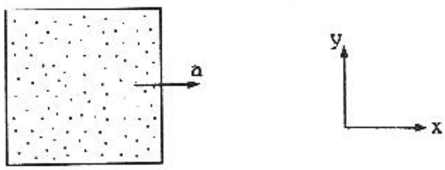
SOLUTIONS

1. (B)

If a fluid (gas or liquid) is accelerated in positive x-direction, then pressure decreases in positive x-direction. Change in pressure has following differential equation—

$$\frac{dP}{dx} = -\rho a$$

where ρ is the density of the fluid. Therefore, pressure is lower in front side.



2. (C)

Speed of sound in an ideal gas is given by

$$V = \sqrt{\frac{\gamma RT}{M}}$$

$\therefore$ $$V \propto \sqrt{\frac{\gamma}{M}}$$ [T is same for both the gases]

$\therefore$ $$\frac{V_{N_2}}{V_{He}} = \sqrt{\frac{\gamma_{N_2}}{\gamma_{H_e}} \cdot \frac{M_{H_e}}{M_{N_2}}}$$

$$= \sqrt{\frac{(7/5)}{(5/3)}\left(\frac{4}{28}\right)}$$

$$= \sqrt{3}/5$$

$\gamma_{N_2} = 7/5$ (Diatomic)

$\gamma_{He} = 5/3$ (Monoatomic)

3. (B)

$$|\text{ average velocity }| = \left|\frac{\text{Displacement}}{\text{time}}\right| = \frac{AB}{\text{time}} = \frac{2}{1} = 2\text{m/s}$$

4. (A)

The charged particle will be accelerated parallel (if it is a positive charge) or antiparallel (if it is a negative charge) to the electric field, i.e., the charged particle will move parallel or antiparallel to electric and magnetic field. Therefore net magnetic force on it will be zero and its path will be a straight line.

5. (C)

Energy is released in a process when total binding energy of the nucleus (= binding energy per nucleon × number of nucleons) is increased or we can say, when total binding energy of products is more than the reactants. By calculation we can see that only in case of option (C), this happens.

Given $W \rightarrow 2Y$

Binding energy of reactants $= 120 \times 7.5 = 900$ MeV

and binding energy of products $= 2(60 \times 8.5) = 1020 \text{ MeV} > 900 \text{ MeV}$

6. (B)

Radius of a nucleus is given by

$$R = R_0 A^{1/3} \qquad (\text{where } R_0 = 1.25 \times 10^{-15} \text{ m})$$

$$= 1.25\, A^{1/3} \times 10^{-15} \text{ m}$$

Here A is the mass number

and mass of the Uranium nucleus will be

$$m \approx A m_p \qquad m_p = \text{mass of proton}$$

$$= A(1.67 \times 10^{-27} \text{ Kg})$$

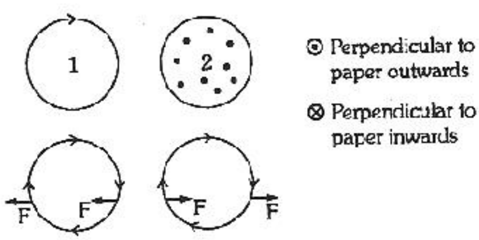
$\therefore$ Density $\rho = \dfrac{\text{mass}}{\text{volume}}$

$$= \frac{m}{\frac{4}{3}\pi R^3} = \frac{A(1.67 \times 10^{-27} \text{ kg})}{A(1.25 \times 10^{-15} \text{ m})^3}$$

or $\rho \approx 2.0 \times 10^{17}\ \text{Kg/m}^3$

7. (C)

For understanding, let us assume that the two loops are lying in the plane of paper as shown. The current in loop 1 will produce • magnetic field in loop 2. Therefore, increase in current in loop 1 will produce an induced current in loop 2 which produces ⊗ magnetic field passing through it i.e. induced current in loop 2 will also be clockwise as shown alongside.



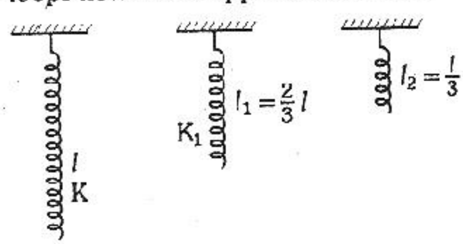
The loops will now repel each other as the currents at the nearest and farthest points of the two loops flow in the opposite directions.

8. (B)

$$l_1 = 2l_2$$

$$l_1 = \frac{2}{3} l$$

Force constant $K \propto \dfrac{1}{\text{length of spring}}$



$$K_1 = \frac{3}{2} K$$

9. (B)

Atomic number of Neon is 10.

By the emission of two α-particles, atomic number will be reduced by 4. Therefore, atomic number of the unknown element will be

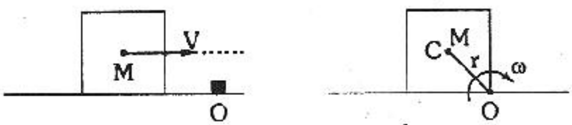
$$Z = 10 - 4$$
$$= 6$$

Similarly mass number of the unknown element will be

$$A = 22 - 2 \times 4$$
$$= 14$$

∴ Unknown nucleus is carbon (A = 14, Z = 6)

10. (A)



$$r = \sqrt{2}\,\frac{a}{2} \quad \text{or} \quad r^2 = \frac{a^2}{2}$$

Net torque about O is zero.

Therefore, angular momentum (L) about point O will be conserved

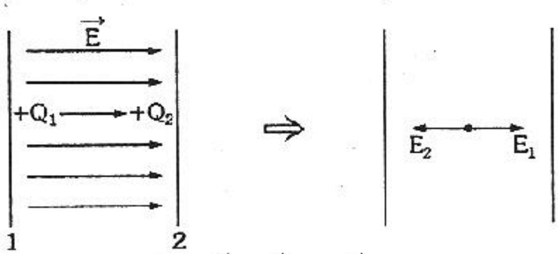
or
$$L_i = L_f$$
$$MV\left(\frac{a}{2}\right) = I_0\,\omega$$
$$= (I_{com} + Mr^2)\,\omega$$
$$= \left\{\left(\frac{Ma^2}{6}\right) + M\left(\frac{a^2}{2}\right)\right\}\omega$$
$$= \frac{2}{3}Ma^2\,\omega$$

∴
$$\omega = \frac{3V}{4a}$$

11. (D)

Diffraction is obtained when the slit width is of the order of wavelength of light (or any electromagnatic wave) used. Here wavelength of X rays (1-100 Å) < < slit width (0.6 mm). Therefore no diffraction pattern will be observed.

12. (D)



Electric field within the plates $\vec{E} = \vec{E}_{Q_1} + \vec{E}_{Q_2}$

or
$$E = E_1 - E_2$$
$$= \frac{Q_1}{2A \epsilon_0} - \frac{Q_2}{2A \epsilon_0}$$
$$E = \frac{Q_1 - Q_2}{2A \epsilon_0}$$

∴ Potential difference between the plates

$$V_A - V_B = E \cdot d = \left(\frac{Q_1 - Q_2}{2A \epsilon_0}\right) d$$
$$= \frac{Q_1 - Q_2}{2\left(\frac{A \epsilon_0}{d}\right)}$$
$$= \frac{Q_1 - Q_2}{2C}$$

13. (A)

Locus of equal path difference are the lines running parallel to the axis of the cylinder. Hence straight fringes are obtained.

→ *Circular rings (also called Newton's rings) are observed in interference pattern when a plano-convex lens of large focal length is placed with its convex surface in contact with a plane glass plate because locus of equal path difference in this case is a circle.*

14. (D)

The current-time (i–t) equation in L-R circuit is given by [Growth of current in L–R circuit]

$$i = i_0 (1 - e^{-t/\tau_L}) \quad \ldots(1)$$

where $i_0 = \frac{V}{R} = \frac{12}{6} = 2\ A$

and $\tau_L = \frac{L}{R} = \frac{8.4 \times 10^{-3}}{6} = 1.4 \times 10^{-3}\ s$

and $i = 1A$ (given)

$t = ?$

Substituting these values in equation (1), we get

$$t = 0.97 \times 10^{-3}\ s$$

or $t = 0.97$ ms

$t \approx 1$ ms

15. (D)

When S_3 is closed, due to attraction with opposite charge, no flow of charge takes place through S_3. Therefore, potential difference across capacitor plates remains unchanged or $V_1 = 30$ V and $V_2 = 20$ V.

Alternate Solution

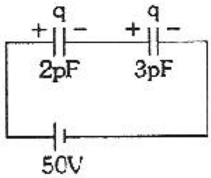
Charges on the capacitors are—

$$q_1 = (30)(2) = 60\ pC \text{ and}$$

$$q_2 = (20)(3) = 60 \text{ pC}$$

or $$q_1 = q_2 = q \text{ (say)}$$

The situation is similar as the two capacitors in series are first charged with a battery of emf 50V and then disconnceted.



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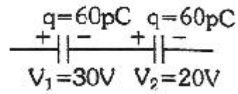


$\therefore$ When S_3 is closed, $V_1 = 30$ V and $V_2 = 20$ V

16. (A)

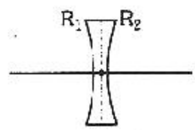
$$R_1 = -R, R_2 = +R, \mu_g = 1.5 \text{ and } \mu_m = 1.75$$

$\therefore$ $$\frac{1}{f} = \left(\frac{\mu_g}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Substituting the values, we have

$$\frac{1}{f} = \left(\frac{1.5}{1.75} - 1\right)\left(\frac{1}{-R} - \frac{1}{R}\right)$$

$$= \frac{1}{3.5\,R}$$



$\therefore$ $$f = +3.5\,R$$

Therefore, in the medium it will behave like a convergent lens of focal length 3.5 R. It can be understood as, $\mu_m > \mu_g$, the lens will change its behaviour.

17. (D)

Internal energy of n moles of an ideal gas at temperature T is given by—

$$U = n\left(\frac{f}{2} RT\right)$$

where f = degrees of freedom.

= 5 for O_2 and 3 for Ar

Hence

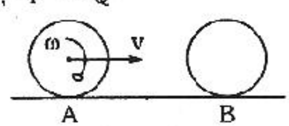
$$U = U_{O_2} + U_{Ar}$$

$$= 2\left(\frac{5}{2} RT\right) + 4\left(\frac{3}{2} RT\right) = 11\, RT$$

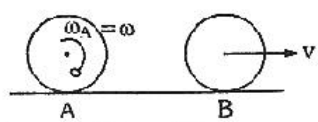
18. (A)

As there is no change in the reading of galvanometer with switch S open or closed. It implies that bridge is balanced. Current through S is zero and $I_R = I_G$, $I_P = I_Q$.

19. (C)



Before collision



After collision

Since it is head on elastic collision between two identical balls, they will exchange their linear velocities i.e. A comes to rest and B starts moving with linear velocity v. As there is no friction anywhere, torque on both the spheres about their centre of mass is zero and their angular velocities remain unchanged. Therefore $\omega_A = \omega$ and $\omega_B = 0$.

20. (B)

Since the wavelength (λ) is increasing, we can say that the galaxy is receding. Doppler effect can be given by—

$$\lambda' = \lambda \sqrt{\frac{c+v}{c-v}} \qquad \ldots(1)$$

or
$$706 = 656 \frac{\sqrt{c+v}}{c-v}$$

or
$$\frac{c+v}{c-v} = \left(\frac{706}{656}\right)^2 = 1.16$$

$\therefore$
$$c + v = 1.16\,c - 1.16\,v$$

$\therefore$
$$v = \frac{0.16\,c}{2.16}$$

$$= \frac{0.16 \times 3.0 \times 10^8}{2.16}\ \text{m/s}$$

$$= 0.22 \times 10^8\ \text{m/s}$$

$$v \approx 2.2 \times 10^7\ \text{m/s}$$

If we take the approximation then equation (1) can be written as—

$$\Delta\lambda = \lambda\left(\frac{v}{c}\right) \qquad \ldots(2)$$

From here
$$v = \left(\frac{\Delta\lambda}{\lambda}\right).c$$

$$= \left(\frac{706-656}{656}\right)(3 \times 10^8)\ \text{m/s}$$

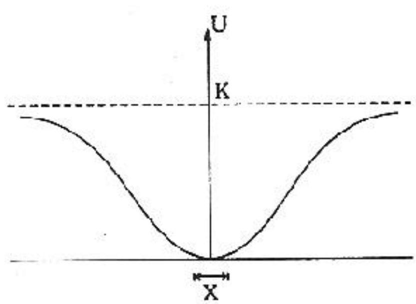
$$v = 0.23 \times 10^8\ \text{m/s}$$

which is almost equal to the previous answer. So we may use equation (2) also.

21. (D)

$$U(x) = K(1 - e^{-x^2})$$

It is an exponentially increasing graph of potential energy (U) with x^2. Therefore U versus x graph will be as shown



From the graph it is clear that at origin—

Potential energy U is minimum (therefore, kinetic energy will be maximum) and force acting on the particle is also zero because $F = \frac{-dU}{dx} = -(\text{slope of } U - x \text{ graph}) = 0$.

Therefore, origin is the stable

equilibrium position. Hence particle will oscillate simple harmonically about $x = 0$ for small displacements. Therefore, correct option is (D).

(A), (B) and (C) options are wrong due to following reasons.

(A) At equilibrium position $F = \frac{-dU}{dx} = 0$ i.e. slope of U–X graph should be zero and from the graph we can see that slope is zero at $x = 0$ and $x = \pm \infty$. Now among these equilibriums stable equilibrium position is that where U is minimum (Here $x = 0$). Unstable equilibrium position is that where U is maximum (Here none).

Neutral equilibrium position is that where U is constant (Here $x = \pm \infty$). Therefore, option (A) is wrong.

(B) For any finite non-zero value of x, force is directed towards the origin, because origin is in stable equilibrium position. Therefore, option (B) is incorrect.

(C) At origin, potential energy is minimum, hence kinetic energy will be maximum. Therefore, option (C) is also wrong.

22. (C)

From law of conservation of momentum,

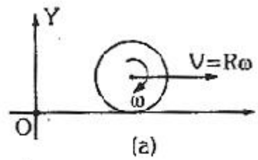
$$P_1 = P_2 \quad \text{(in opposite directions)}$$

Now de-Broglie wavelength is given by

$$\lambda = \frac{h}{P} \quad h = \text{Planck's constant}$$

Since momentum (P) of both the particles is equal, therefore $\lambda_1 = \lambda_2$

or $\lambda_1/\lambda_2 = 1$



(a)

23. (D)

Total magnetic flux passing through whole of the X-Y plane will be zero, because magnetic lines form a closed loop. So as many lines will move in –Z direction same will return to + Z direction from the X–Y plane.

24. (A)

Both the beta rays and the cathode rays are made up of electrons. So only option (A) is correct.

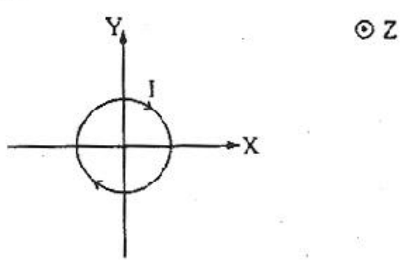
(B) Gamma rays are electromagnetic waves.

(C) Alpha particles are doubly ionized helium atoms, and

(D) Protons and Neutrons have approximately the same mass.

Therefore (B), (C) and (D) are wrong options.

25. (C)



From the theorem—

$$\vec{L}_O = \vec{L}_{com} + M(\vec{r} \times \vec{V}) \quad \ldots(1)$$

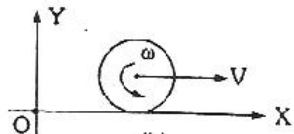
We may write

Angular momentum about O = Angular momentum about COM + Angular momentum of COM about origin

$\therefore \quad L_0 = I\omega + MRV$

$$= \frac{1}{2}MR^2\omega + MR(R\omega)$$

$$= \frac{3}{2}MR^2\omega$$



(b)

Note that in this case both the terms in equation (1) i.e. $\vec{L}_{com}$ and $M(\vec{r} \times \vec{V})$ have the same direction ⊗. That is why we have used $L_0 = I\omega + MRV$. We will use $L_0 = I\omega \sim MRV$ if they are in opposite directions as shown in figure (b).

26. **(A, B, C)**

$$x = a\cos pt \Rightarrow \cos(pt) = \frac{x}{a} \qquad \ldots(1)$$

$$y = b\sin pt \Rightarrow \sin(pt) = \frac{y}{b} \qquad \ldots(2)$$

Squaring and adding (1) and (2), we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Therefore, path of the particle is in ellipse. Hence option (A) is correct.

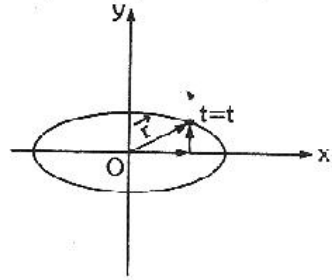
From the given equations we can find—

$$\frac{dx}{dt} = v_x = -a\,p\sin pt$$

$$\frac{d^2x}{dt^2} = a_x = -ap^2\cos pt$$

$$\frac{dy}{dt} = v_y = bp\cos pt \text{ and}$$

$$\frac{d^2y}{dt^2} = a_y = -bp^2\sin pt$$



At time $\quad t = \pi/2p$ or $pt = \pi/2$

a_x and v_y become zero (because $\cos \pi/2 = 0$) only v_x and a_y are left,

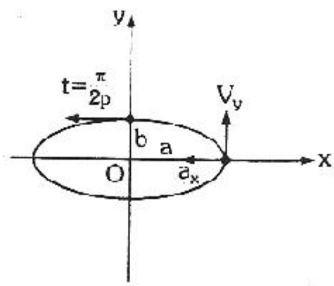
or we can say that velocity is along negative x-axis and acceleration along y-axis.

Hence at $t = \pi/2p$, velocity and acceleration of the particle are normal to each other. So option (B) is also correct.

At $t = t$, position of the particle

$$\vec{r}(t) = x\hat{i} - y\hat{j}$$

$$= a\cos pt\ \hat{i} + b\sin pt\ \hat{j}$$



and acceleration of the particle is

$$\vec{a}(t) = a_x\hat{i} + a_y\hat{j}$$

$$= -p^2[a\cos pt\hat{i} + b\sin pt\ \hat{j}]$$

$$= -p^2[x\hat{i} + y\hat{j}] = -p^2\vec{r}(t)$$

Therefore acceleration of the particle is always directed towards origin.

Hence option (C) is also correct.

$t = 0$

$y = 0 - v_x = a_y$

$x = a$

$v_y = bp$ and

$a_x = -ap^2$

At $t = 0$, particle is at $(a, 0)$ and at $t = p/2p$, particle is at $(0, b)$. Therefore, the distance covered is one-fourth of the elliptical path not a. Hence option (D) is wrong.

27. (C)

$$(t_{1/2})_x = (t_{mean})_y$$

or $$\frac{0.693}{\lambda_x} = \frac{1}{\lambda_y}$$

$\therefore$ $$\lambda_x = 0.693\,\lambda_y$$
$$\lambda_x < \lambda_y$$

or Rate of decay $= \lambda N$

Initially number of atoms (N) of both are equal but since $\lambda_y > \lambda_x$, therefore, y will decay at a faster rate than x.

28. (C, D)

Under electrostatic condition, all points lying on the conductor are in same potential. Therefore, potential at A – potential at B. Hence option (C) is correct. From Gauss theorem, total flux through the surface of the cavity will be q/ϵ_0.

→ *Instead of an elliptical cavity, if it would had been a spherical cavity then options (A) and (B) were also correct.*

29. (A, C)

From superposition principle—

$$y = y_1 + y_2 + y_3$$
$$= a \sin \omega t + a \sin(\omega t + 45°) + a \sin(\omega t + 90°)$$
$$= a \{\sin \omega t + \sin(\omega t + 90°)\} + a \sin(\omega t + 45°)$$
$$= 2a \sin(\omega t + 45°) \cos 45° + a \sin(\omega t + 45°)$$
$$= (\sqrt{2} + 1)\, a \sin(\omega t + 45°)$$
$$= A \sin(\omega t + 45°)$$

Therefore, resultant motion is simple harmonic of amplitude

$$A = (\sqrt{2} + 1)\, a$$

and which differ in phase by 45° relative to the first.

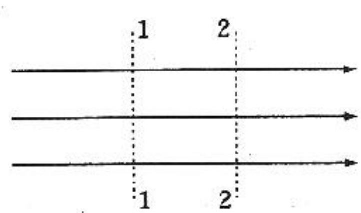
Energy in SHM $\propto$ (amplitude)^2 $$E = \frac{1}{2} m A^2 \omega^2$$

$\therefore$ $$\frac{E_{resultant}}{E_{single}} = \left(\frac{A}{a}\right)^2 = (\sqrt{2}+1)^2 = (3 + 2\sqrt{2})$$

$\therefore$ $$E_{resultant} = (3 + 2\sqrt{2})\, E_{single}$$

30. (A, C, D)

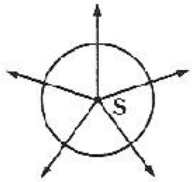
For a plane wave intensity (energy crossing per unit area per unit time) is constant at all points.

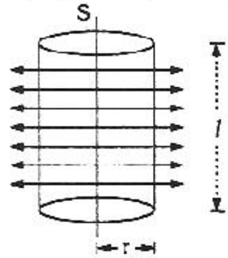


$I_{11} = I_{22}$

But for a spherical wave, intensity at a distance r from a point source of power P (energy transmitted per unit time) is given by

$I = \frac{P}{4\pi r^2}$ or $I \propto \frac{1}{r^2}$



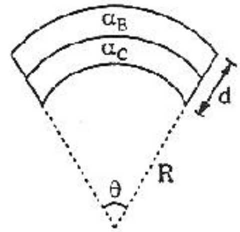


→ *For a line source* $I \propto \frac{1}{r}$

because $I = \frac{P}{\pi r l}$

31. (B, D)

Let l_0 be the initial length of each strip before heating.
Length after heating will be—



$$l_B = l_0 (1 + \alpha_B \Delta T) = (R + d)\,\theta \text{ and}$$
$$l_C = l_0 (1 + \alpha_C \Delta T) = R\,\theta$$

$\therefore$ $$\frac{R + d}{R} = \left(\frac{1 + \alpha_B \Delta T}{1 + \alpha_C \Delta T}\right)$$

$\therefore$ $$1 + \frac{d}{R} = 1 + (\alpha_B - \alpha_C)\Delta T$$ [From binomial expansion]

$$R = \frac{d}{(\alpha_B - \alpha_C)\Delta T}$$

or $$R \propto \frac{1}{\Delta T} \text{ and}$$

$$\propto \frac{1}{|\alpha_B - \alpha_C|}$$

32. (A, B, D)

At 0 K, a semiconductor becomes a perfect insulator. Therefore at 0 K, if some potential difference is applied across an insulator or a semiconductor, current is zero. But a conductor will become a superconducter at 0 K. Therefore, current will be infinite. In reverse biasing at 300 K through a p-n junction diode, a small finite current flows due to minority charge carriers.

33. (B, C, D)

The shape of pulse at x = 0 and t = 0 would be as shown, in figure (a)
$y(0, 0) = \frac{0.8}{5} = 0.16$ m

From the figure it is clear that $y_{max} = 0.16$ m
Pulse will be symmetric (Symmetry is checked about y_{max}) if

$$\text{At } t = 0;\ y(x) = y(-x)$$

From the given equation

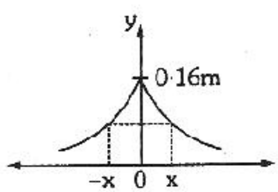
$$y(x) = \frac{0.8}{16x^2 + 5}$$

and

$$y(-x) = \frac{0.8}{16x^2 + 5}$$

at t = 0

or $y(x) = y(-x)$
Therefore pulse is symmetric.

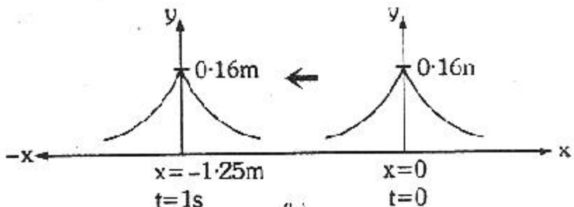


t=0
(a)

Speed of pulse →

At $t = 1s$ and $x = -1.25$ m

value of y is again 0.16 m. i.e. pulse has travelled a distance of 1.25 m in 1 second in negative x-direction or we can say that the speed of pulse is 1.25 m/s and it is travelling in negative x-direction. Therefore, it will travel a distance of 2.5 m in 2 seconds. The above statement can be better understood from figure (b).



(b)

Alternate method

If equation of a wave pulse is

$$y = f(ax \pm bt)$$

the speed of wave is $\frac{b}{a}$ in negative x direction for $y = f(ax + bt)$ and positive x direction for $y = f(ax - bt)$. Comparing this from given equation we can find that speed of wave is $5/4 = 1.25$ m/s and it is travelling in negative x-direction.

34. (A, B, C, D)

In case of sound wave, y can represent pressure and displacement, while in case of an electromagnetic wave it represents electric and magnetic fields.

→ *In general, y is any general physical quantity which is made to oscillate at one place and these oscillations are propagated to other places also.*

35. (A, B, C)

Standing waves can be produced only when two similar type of waves (same frequency and speed, but amplitude may be different) travel in opposite directions.